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Method for measuring K_{11}/K_{33} and $\varepsilon_{\|}/\varepsilon_{\perp}$ ratios of nematic liquid crystals with negative dielectric anisotropy

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For any nematic liquid crystal having a negative dielectric anisotropy ε_α, in a homeotropic texture sandwiched between two parallel plates with a sufficiently large distance L between them, and under a square external electric field excitation $\mathbf{E}_0(t)=\pm\mathbf{E}_0$ and a static magnetic field $\mathbf{H}$ perpendicular to the plates, the applied external voltage V at the threshold of the electrohydrodynamic instability and the square S^2 of the quantity $S=qL/\pi$ are linear functions of the Fréedericksz transition voltage V_F. Here π/L is equal to the wave number of the distortion along the Z-axis perpendicular to the walls and q is the wave number of distortion along any X-direction perpendicular to the Z-axis. S^2 is also a linear function of voltage V. The two lines $V_F(V_\mathrm{F})$ and $V(V_\mathrm{F})$ intersect at a value of voltage V_0 such that $V=V_F=V_0$, which depends only on frequency. With experimental measurements around the point V_0, one may compute the ratio of the elastic constants, K_{11}/K_{33} where K_{11} and K_{33} are the splay and bend elastic constants, respectively, and by combining the slopes of the two lines $S^2(V_\mathrm{F})$ and $S^2(V)$, one may obtain an estimate of the ratio of the dielectric constants $\varepsilon_{\|}/\varepsilon_{\perp}$.

1. Introduction

A nematic liquid crystal with negative dielectric anisotropy $\varepsilon_\alpha=\varepsilon_{\|}-\varepsilon_{\perp}$ is placed between two parallel plates with a distance L between them. The initial direction h_0 of the director is parallel to the Z-axis and perpendicular to the plates. A static and homogenous external magnetic field $\mathbf{H}$ is applied parallel to the Z-axis. Under an external electric field $\mathbf{E}_0(t)$, the director h makes an angle θ with respect to the initial direction on an arbitrary plane XOZ (figure 1).

In the conditions of the Fréedericksz transition (FT) the angle θ, which is very small, and also the director h, depend only on the z component and we use the equation

$$V_F^2=A+B\mathbf{H}^2 \tag{1}$$

where $V_\mathrm{F}=\mathbf{E}_0L$ is the FT voltage, $A=4\pi^3K_{33}/-\varepsilon_\mathrm{a}$, and $B=4\pi\chi_\mathrm{a}L^2/-\varepsilon_\mathrm{a}$, K_{33} is the bend elastic constant and $\chi_\alpha=\chi_{\|}-\chi_{\perp}$ is the magnetic anisotropy.

All the physical quantities or functions involved in the various equations depend on the angle θ, which is very small [1], particularly around the value V_0, and we can retain only those terms which are linear in θ. We accept also that all the physical quantities or functions are periodic with respect to x and z [2].

Taking into account the distortion energy [3], force equations of liquids and the viscous stress tensor [4, 5], the forces and effects due magnetic [6] and electric fields [7], the torque equation [8, 9], the theory for distortions of Dubois *et al.* [10], and the linear theory of electro-hydrodynamic instabilities (EHDI) [11] modified to homeotropic textures of nematics with negative dielectric anisotropy and under the conditions described, one finds two equations relating the charge density ρ and the curvature $\psi=\partial\theta/\partial x$:

$$\dot{\rho}+\frac{1}{T_q}\rho+b_1\mathbf{E_0}\psi=0\ \left(\dot{\rho}=\frac{\partial\rho}{\partial t}\right) \tag{2}$$

$$\dot{\psi}+\frac{1}{T_\psi}\psi+b_2\mathbf{E_0}\rho=0\ \left(\dot{\psi}=\frac{\partial\psi}{\partial t}\right) \tag{3}$$

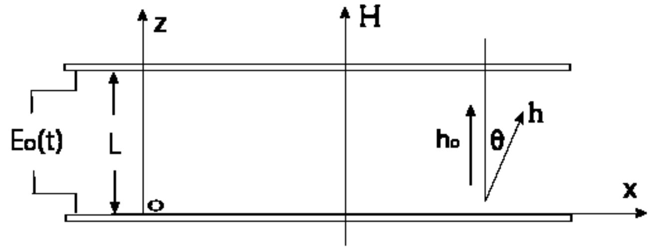

Figure 1. The conditions of the nematic layer.

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where

$$T_q = \frac{\varepsilon_\perp S^2 + \varepsilon_\parallel}{4\pi(\sigma_\perp S^2 + \sigma_\parallel)}$$
$$\frac{1}{T_\psi} = \xi_1 + \xi_2 E_0^2$$
$$b_1 = \frac{\sigma_H(S^2+1)}{\varepsilon_\perp S^2 + \varepsilon_\parallel}, \tag{4}$$
$$b_2 = -\frac{S^2\left[\varepsilon_\alpha \eta + (\varepsilon_\perp S^2 + \varepsilon_\parallel)(\alpha_3 S^2 - \alpha_2)\right]}{(\varepsilon_\perp S^2 + \varepsilon_\parallel)\left[\gamma_1 \eta - (\alpha_3 S^2 - \alpha_2)^2\right]}$$

Here $\sigma_\parallel$ and $\sigma_\perp$ are the conduction constants, $\sigma_H = \varepsilon_\perp \sigma_\parallel - \varepsilon_\parallel \sigma_\perp$, and $S = qL/\pi$, where q is the distortion wave number along the X-axis and

$$\xi_1 = \frac{\chi_\alpha H^2 + (K_{11} S^2 + K_{33})\pi^2 / L^2}{\gamma_1 - (\alpha_3 S^2 - \alpha_2)^2 / \eta} \tag{5}$$

$$\xi_2 = \frac{\varepsilon_\alpha \varepsilon_\parallel (S^2+1)}{4\pi(\varepsilon_\perp S^2 + \varepsilon_\parallel)\left[\gamma_1 - (\alpha_3 S^2 - \alpha_2)^2 / \eta\right]} \tag{6}$$

for

$$\eta = \eta_1 + (\eta_1 + \eta_2 + \alpha_1) S^2 + \eta_2 S^4, \; \eta_1 = \frac{1}{2}(\alpha_4 + \alpha_5 - \alpha_2),$$
$$\eta_2 = \frac{1}{2}(\alpha_3 + \alpha_4 + \alpha_6),$$

$\gamma_1 = \alpha_2 - \alpha_3$, and $\alpha_i (i = 1,2,3,4,5,6)$ are the Leslie coefficients.

These equations are more amenable to calculations, needed for the problem, than the more rigorous formulations of EHDI theory [12–16]. Those formulations also extend to conditions of non-linear behaviour of the nematic phase, which is not important in the present work, particularly around the point $V = V_F = V_0$ where the angle θ is very small.

2. Theoretical approach

For a key to symbols used in this discussion see table 1. Generally $\varepsilon_\parallel\ (S^2+1)/(\varepsilon_\perp S^2 + \varepsilon_\parallel) \sim 1$ and for $S \geqslant 4$ $(\alpha_3 S^2 - \alpha_2)^2/\eta \ll \gamma_1$ where $\gamma_1 \sim 1$. From equations (4), (5) and (6) we obtain

$$\frac{1}{T_\psi} \chi_\alpha H^2 + \frac{(K_{11} S^2 + K_{33})\pi^2}{L^2} - \frac{-\varepsilon_\alpha}{4\pi} E_0^2.$$

The term $1/T_\psi$ is sufficiently small because the torque due to the electric field is comparable to the sum of torques due to the magnetic field and to the distortion. Thus, the bend relaxation time for molecular orientation T_ψ is much larger than the frequency f of $\mathbf{E}_0(t)$, and the curvature ψ cannot follow the orientation of the oscillating field $\mathbf{E}_0(t)$. Therefore, ψ may be considered as being static and constant with respect to time t. Thus we set $\dot{\psi} = \partial\psi/\partial t = 0$ and

Table 1. Symbols used in this paper.

Symbol	Designation
L	Sample thickness
$\mathbf{E}_0(t)$	External electric field excitation
$V(t) = \mathbf{E}_0(t)L$	Applied external voltage at threshold of the electrohydrodynamic instability (EHDI)
H	External magnetic field
V_F	Fréedericksz transition voltage
A, B	Coefficients of Fréedericksz transition equation $V_F{}^2 = A + B\mathbf{H}^2$
$\varepsilon_\parallel$, $\varepsilon_\perp$	Dielectric constants
$\varepsilon_\alpha = \varepsilon_\parallel - \varepsilon_\perp$	Dielectric anisotropy
$\sigma_\parallel$, $\sigma_\perp$	Conduction constants
$\chi_\alpha = \chi_\parallel - \chi_\perp$	Magnetic anisotropy
$S = qL/\pi$	q is the wave number of distortion along any X-direction perpendicular to the Z-axis
K_{11}, K_{33}	Splay and bend elastic constants
h_E	Ratio $\varepsilon_\parallel/\varepsilon_\perp$
h_S	Ratio $\sigma_\parallel/\sigma_\perp$
h_K	Ratio K_{11}/K_{33}
f	frequency
$x = \varepsilon_\perp f/\pi\sigma_\perp$	Variable which replaces the frequency
$u(x)$	Function $x \tanh(x^{-1})/[1 - x\tanh(x^{-1})]$
$V(x, V_F)$	Applied external voltage $V(t)$ at threshold of EHDI, which finally depends on variables x, V_F
$S^2(x, V_F)$	Square of $S = qL/\pi$, which depends on variables x, V_F
$\lambda'(x)$, $k'(x)$	Coefficients of linear equation $V(x, V_F) = \lambda'(x) + k'(x)V_F$
$\lambda(x)$, $k(x)$	Coefficients of linear equation $S^2(x, V_F) = \lambda(x) + k(x)V_F$
$t(x)$	Slope of plot $S^2(x, V_F)$ against $V(x, V_F)$
V_0	Intersection point of the lines $V_F(V_F)$ and $V(V_F)$ (where $V = V_F = V_0$)

equation (3) becomes

$$\frac{1}{T_\psi}\psi + b_2\mathbf{E_0}\rho = 0. \qquad (7)$$

A detailed theoretical analysis is given in the appendix. Equation (A11), under the conditions $\alpha/\beta(x)V^2 \ll 1$ and $\gamma(x)/\varepsilon(x)V_F{}^2 \ll 1$ (see appendix), becomes

$$k'(x) \approx \left(\frac{\varepsilon(x)}{\beta(x)}\right)^{\frac{1}{2}}. \qquad (8)$$

For $f=0$ (i.e. $x=0$ and $u(x)=u(0)=0$) and from equation (8) we obtain

$$k'(0) = \left(\frac{\varepsilon(0)}{\beta(0)}\right)^{\frac{1}{2}} = \left(\frac{e_2}{h_2}\right)^{\frac{1}{2}} = \left(\frac{1}{h_S + \frac{\alpha_3 m}{\eta_2}}\right)^{\frac{1}{2}} < 1$$

and for $f\to\infty$ ($x\to\infty$ and $u(x)\to\infty$)

$$k'(\infty) \to \left(\frac{1}{h_E}\right)^{\frac{1}{2}} = \left(\frac{\varepsilon_\perp}{\varepsilon_\parallel}\right)^{\frac{1}{2}} > 1.$$

We can see from the last two relationships, as x (or frequency f) increases, the slope $k'(x)$ of the linear equation (A12) also increases. Because the slope of the line $V_F(V_F)$ is equal to one, the two lines $V_F(V_F)$ and $V(V_F)$ intersect at a value V_0 of the voltage V, which depends only on frequency and increases rapidly as frequency increases. This can be expressed as,

$$V_0(x) = \frac{\lambda'(x)}{1-k'(x)} \approx \frac{1}{y_2(x)} \frac{1}{(\beta(x))^{\frac{1}{2}} - (\varepsilon(x))^{\frac{1}{2}}}. \qquad (9)$$

This intersection takes place when $\beta(x) > \varepsilon(x)$ or $u(x) < (h_2/e_2 - 1)/(1-h_E)$. For values of f equal to a certain frequency f_C (or $x=x_C$), which may be obtained from the relationship $u(x_C) = (h_2/e_2 - 1)/(1-h_E)$, equation (9) gives $V_0(x_C) = \infty$ (or $V_0(x) < 0$ for $f > f_C$) and there is no intersection.

From equation (1) we have $V_0 = \left(A + B\mathbf{H}_0^2\right)^{\frac{1}{2}}$. For any frequency $f < f_C$, on increasing the external voltage from zero, and for values of magnetic field $\mathbf{H} < \mathbf{H}_0$, we first observe the FT and second, the EHDI. For $\mathbf{H} \geqslant \mathbf{H}_0$ only the EHDI is observed (see figures 2–4 for MBBA).

We rewrite equation (A19) as

$$\lambda'(x) = Ah_K\left[k(x)k'(x) + \frac{\lambda(x)(1-k'(x))}{\lambda'(x)}\right] \qquad (10)$$

The quantities $\lambda'(x)$, $k'(x)$, $\lambda(x)$ and $k(x)$ in the linear equations (A12), (A15) (see the appendix) are measurable, as is the quantity A, in equation (1). From equation (10) one may compute the ratio of the elastic

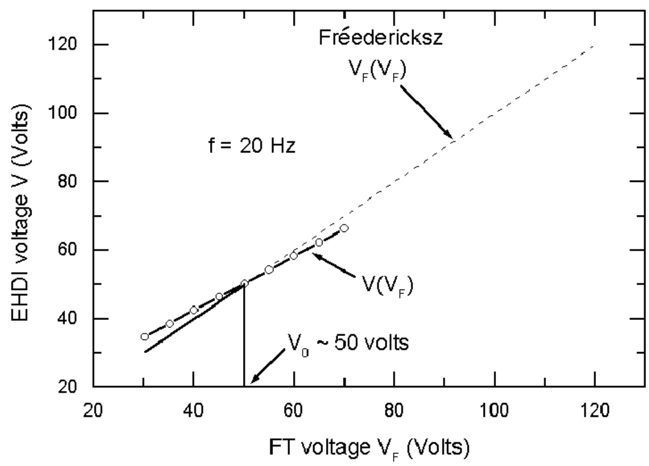

Figure 2. Plot of the threshold V of EHDI vs. FT voltage V_F at a frequency of 20 Hz for MBBA.

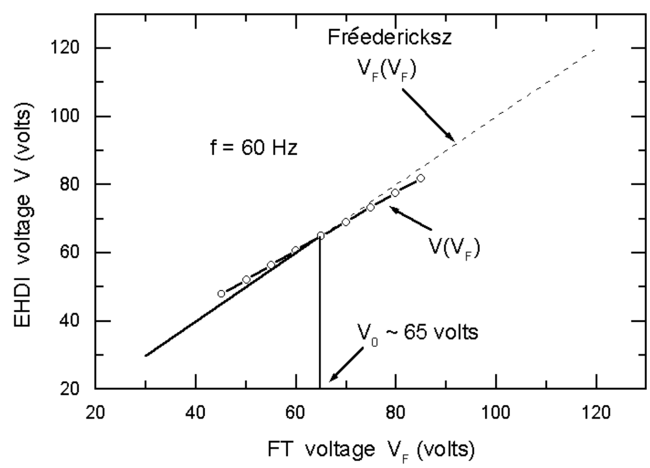

Figure 3. Plot of threshold V of EHDI vs. FT voltage V_F at a frequency of 60 Hz for MBBA.

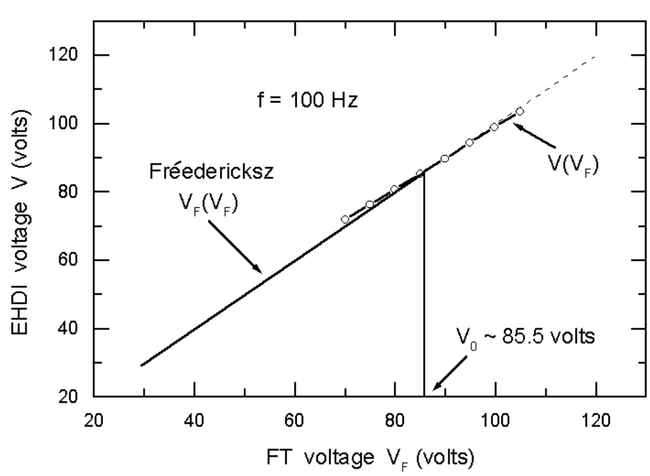

Figure 4. Plot of threshold V of EHDI vs. FT voltage V_F at a frequency of 100 Hz for MBBA.

constants $h_K=K_{11}/K_{33}$ by taking experimental measurements at a single frequency.

We also rewrite equation (A21) of the appendix,

$$\frac{1}{k(x)^2}=h_E\frac{1}{t(x)^2}+C. \tag{11}$$

Here, C is a constant, which depends on the ratios $h_K=K_{11}/K_{33}$ and $h_E=\varepsilon_\parallel/\varepsilon_\perp$, and on the Leslie coefficients $\alpha_i(i=1,2,3,4,5,6)$. As we can see from equation (11), the ratio $h_E=\varepsilon_\parallel/\varepsilon_\perp$ equals the slope of the plot $1/k(x)^2 \to 1/t(x)^2$, where $k(x)$ and $t(x)$ are taken at various (at least two) frequencies.

3. Experimental confirmation

3.1. Experimental set-up

The inner surfaces of the two parallel glass plates are conductive (In_2O_3), and treated with a very thin layer of lecithin which orients the molecules of any nematic liquid crystal (in the present work, MBBA) perpendicular to the plates. A mylar sheet sets the distance between the two plates equal to $L=(172\pm5)\,\mu m=(17.2\pm0.5)\times10^{-5}\,m$. The experiments take place at a temperature of 23–24°C.

A laser beam, polarized normal to the sheet, is passed through the liquid crystal after a complete reflection in a prism (figure 5). The beam leaving the crystal is reflected again in a second prism and falls on a screen, at a distance from the centre of the magnet, through a polarizer whose optical axis is normal to the polarization plane of the beam. A voltage power supply for the square electric excitation of MBBA and a current power supply (with ammeter) for the magnet are used. The intensity **H** of the magnetic field is controlled as a function of current. With no external electric excitation, one sees nothing on the screen. In conditions of FT, a single dot of light appears. In the EHDI at voltage threshold V, the texture behaves like a 'diffraction grating' and concentric circles appear (figure 6).

3.2. Measuring the quantity $S=qL/\pi$

Because of the boundary conditions (the molecules on the plates are normal to them), the distortion wavelength λ_z along the Z-axis is $\lambda_z \sim 2L$ and the corresponding wave number is equal to π/L.

Let $\lambda_X=2\pi/q$ be the distortion wavelength along the X-axis. From the formula $d\sin\phi=\lambda$, where λ is the wavelength of the laser beam and d the 'diffraction grating constant', we have $d=\lambda_X=2\pi/q$ and

$$\sin\phi=\frac{\lambda q}{2\pi}. \tag{12}$$

If the distance between the texture and the screen is

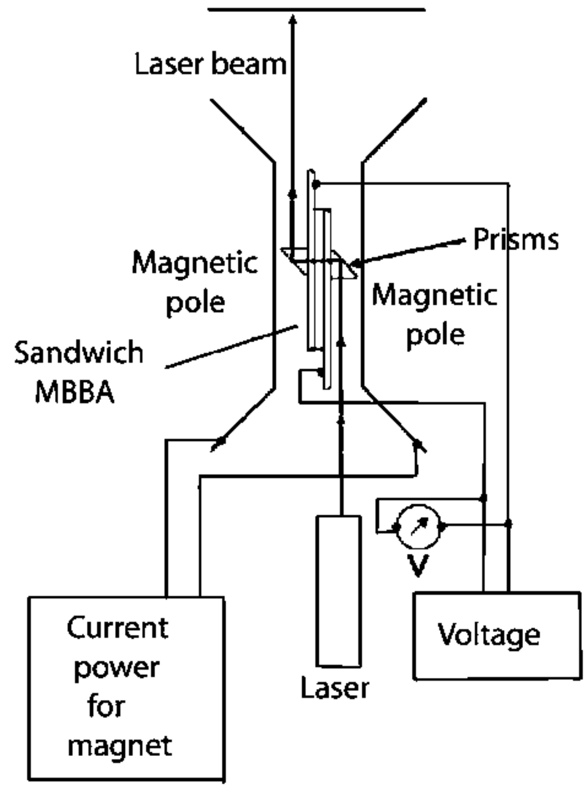

Figure 5. The experimental set-up.

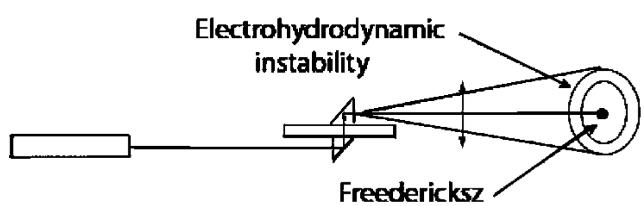

Figure 6. The EHDI 'diffraction grating'.

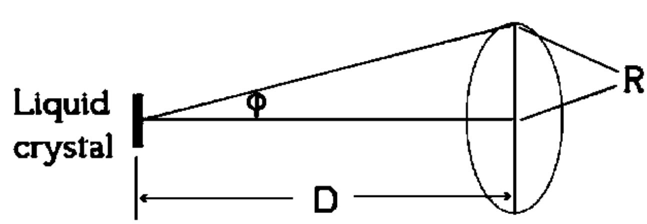

Figure 7. Measuring the ratio $S=k_X/k_Z$.

Table 2. Experimental FT results. The square $\mathbf{H}^2$ of the magnetic field intensity $\mathbf{H}$(kG) and the respective values of the square $V_F{}^2$ of FT voltage V_F(V).

H	$\mathbf{H}^2$	V_F	$V_F{}^2$
0.0	0.00	4.3	18.49
0.5	0.25	6.6	43.56
1.0	1.00	10.9	118.81
1.5	2.25	15.6	243.36
2.0	4.00	20.4	416.16
2.5	6.25	25.3	640.09
3.0	9.00	30.3	918.09
3.5	12.25	35.2	1239.04
4.0	16.00	40.1	1608.01
4.5	20.25	45.1	2034.01
5.0	25.00	50.1	2510.01
5.5	30.25	55.0	3025.00
6.0	36.00	60.0	3600.00

equal to D, one finds (figure 7)

$$\sin\phi = R\Big/\left(D^2+R^2\right)^{1/2} = R/D\ (R << D). \quad (13)$$

Finally, combining equations (12) and (13), we obtain

$$S = \frac{qL}{\pi} = \frac{L}{D\lambda}2R.$$

Constants L, D, λ are known; by measuring the circle diameter $2R$ on the screen we can compute the quantity $S=qL/\pi$ or the square S^2. Thus, for any value of the magnetic field $\mathbf{H}$, one can find experimentally the corresponding threshold voltage V of EHDI, and S from the circle diameter.

3.3. *Experimental measurements*

Because the entire theory is based on formulae expressed in the cgs unit system, the constants A, B of the equation $V_F{}^2=A+B\mathbf{H}^2$ in cgs are given by the relationships $A=4\pi^3K_{33}/-\varepsilon_a$ and $B=4\pi\chi_\alpha L^2/-\varepsilon_\alpha$, where V_F is expressed in statvolts, $\mathbf{H}$ in gauss, K_{33} in dynes, and L in centimetres. Usually, for liquid crystals, V_F is expressed in volts, $\mathbf{H}$ in kgauss (10^3 gauss), K_{33} in dynes, and L in micrometers (10^{-6} m); the constants A, B become

$$A = 36\pi^3K_{33}10^4\big/-\varepsilon_a \text{ and } B = 36\pi\chi_\alpha L^2 10^2\big/-\varepsilon_\alpha$$

The experimental results for the FT for MBBA (table 2) were measured at 60 Hz. From the plot of $V_F{}^2$ against $\mathbf{H}^2$ (figure 8) we calculate the parameters A, B: $A=(19.32\pm0.88)$ volts^2 and $B=(99.47\pm0.05)$ volts^2 kgauss^{-2}. It is easy now to compute, for any value of $\mathbf{H}$, the respective value of V_F, independent of whether the FT is seen or not. In the present work, V and S^2 are expressed as functions of the expression $V_F=(A+B\mathbf{H}^2)^{1/2}$ and not as functions of experimental values V_F of FT. Table 3 gives the experimental results in EHDI in various frequencies around the point V_0.

From table 4, the results of the measurable quantities give a mean value for the ratio $h_K=K_{11}/K_{33}$: $K_{11}/K_{33}=0.692\pm0.003$ or $K_{33}/K_{11}=1.446\pm0.006$.

This result is between the experimental limits $K_{33}/K_{11}=1.11$–1.6 of different authors [17], and it is in good agreement with the value $K_{33}/K_{11}=1.4\pm0.2$ [18].

From the slopes $k(x)$ and $t(x)$ (table 4) of the plots S^2 against V_F and S^2 against V, and the plot $1/k(x)^2$ against $1/t(x)^2$ (figure 9), one may obtain the value of the ratio $h_E=\varepsilon_{\|}/\varepsilon_\perp$, which for MBBA was found to be $h_E=\varepsilon_{\|}/\varepsilon_\perp=0.885\pm0.008$. This result is between the values $\varepsilon_{\|}/\varepsilon_\perp=4.72/5.25=0.899$ and $\varepsilon_{\|}/\varepsilon_\perp=4.7/5.4=0.87$, taken from [19] and [20], respectively.

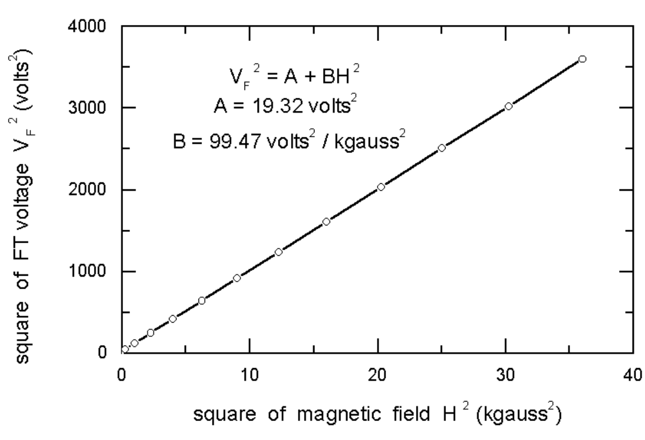

Figure 8. The plot $V_F{}^2$ against H^2 of FT voltage $\mathrm{V_F}^2$ for any respective value $\mathbf{H}^2$ of magnetic field in FT transition for MBBA.

Table 3. Experimental results of the threshold voltage V(V) of the EHDI, and the square S^2 of ratio $S=qL/\pi$, for different values of the magnetic field **H**(kG) and FT voltage V_F(V), around the point V_0, at five frequencies.

		20 Hz		40 Hz		60 Hz		80 Hz		100 Hz	
H	V_F	V	S^2	V	S^2	V	S^2	V	S^2	V	S^2
3.0	30.2	34.5	30.91								
3.5	35.2	38.5	35.32								
4.0	40.1	42.5	39.34	43.1	36.97						
4.5	45.1	46.3	43.57	47.1	41.07	47.9	37.98				
5.0	50.1	50.2	48.02	51.2	45.03	52.1	42.13				
5.5	55.0	54.2	52.28	55.3	49.16	56.3	45.76	57.3	42.13		
6.0	60.0	58.2	56.73	59.4	53.48	60.6	49.55	61.7	45.76		
6.5	65.0	62.2	60.93	63.5	57.56	64.8	53.48	66.1	49.55		
7.0	70.0	66.2	65.28	67.6	61.36	69.0	57.56	70.4	53.08	71.8	48.40
7.5	74.9			71.7	65.72	73.3	61.36	74.8	56.73	76.3	51.89
8.0	79.9					77.5	65.28	79.2	60.08	80.8	55.09
8.5	84.9					81.8	68.89	83.6	63.96	85.2	58.39
9.0	89.9							87.9	67.51	89.7	61.79
9.5	94.8							92.3	71.16	94.3	65.28
10.0	99.8									98.9	68.87
10.5	104.8									103.4	72.09

Table 4. Results of the measurable quantities. The measurable quantities $k'(x)$, $\lambda'(x)$(V), $k(x)$(V^{-1}), $\lambda(x)$, $t(x)$(V^{-1}), $1/k(x)^2$(V^2), $1/t(x)^2$(V^2), and the ratio k_{11}/k_{33} from equation (28) at five frequencies (Hz).

f	k'	λ'	k	λ	t	$1/k^2$	$1/t^2$	k_{11}/k_{33}
20	0.795	10.491	0.864	4.763	1.087	1.340	0.846	0.696
40	0.823	10.040	0.825	3.847	1.003	1.469	0.994	0.696
60	0.852	9.449	0.778	2.961	0.914	1.652	1.197	0.690
80	0.879	8.962	0.727	2.159	0.828	1.892	1.459	0.694
100	0.908	8.248	0.681	0.687	0.751	2.156	1.773	0.682

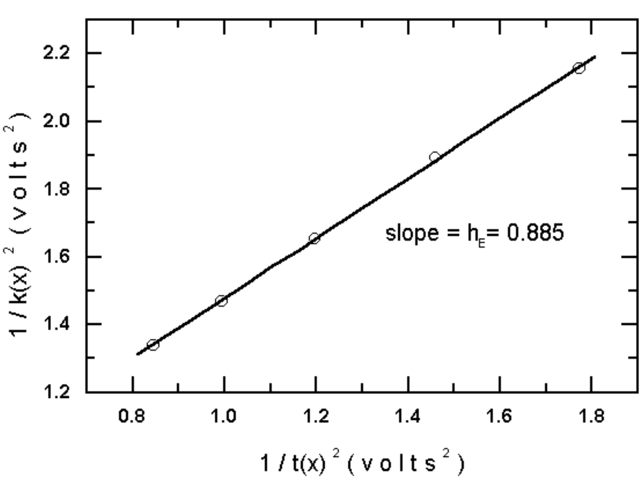

Figure 9. The plot $1/k(x)^2$ against $1/t(x)^2$ of slopes $k(x)$ and $t(x)$ from which we may obtain the ratio $h_E=\varepsilon_\parallel/\varepsilon_\perp$.

If we know the magnetic anisotropy $\chi_\alpha=\chi_\parallel-\chi_\perp$, from the coefficients A and B of equation (1) we can calculate the values of K_{11}, K_{33}, $\varepsilon_\parallel$ and $\varepsilon_\perp$. In addition, using the same experimental measurements, one can obtain the ratios $\varepsilon_\perp/\sigma_\perp$ (from the variable $x=\varepsilon_\perp f/\pi\sigma_\perp$) and $\sigma_\parallel/\sigma_\perp$, but the corresponding theoretical analysis is somewhat more involved.

4. Conclusion

This work provides a method for measuring the ratios K_{11}/K_{33} and $\varepsilon_\parallel/\varepsilon_\perp$, i.e. ratios of elastic and dielectric constants, respectively, for any nematic liquid crystal with a negative dielectric anisotropy. Except for experimental results at the FT for obtaining the factor A of equation (1), one only needs measurements at a single frequency, around the point V_0, in order to calculate the quantities $\lambda'(x)$, $k'(x)$, $\lambda(x)$, and $k(x)$ from plots of V against V_F and S^2 against V_F, and finally from equation (10) the ratio K_{11}/K_{33}. One also needs measurements at a number of frequencies (at least two) in order to calculate the slopes $k(x)$ and $t(x)$ of the plots S^2 against V_F and S^2 against V for each frequency, and finally the slope (which equals the ratio $\varepsilon_{\parallel}/\varepsilon_{\perp}$) of the plot $1/k(x)^2$ against $1/t(x)^2$ (equation 11). The only disadvantage of this method is that it requires as exact measurements as possible, and for any value of magnetic field we must take the FT threshold V_F several times (8–10 in the present work) in order to obtain a reliable mean value. The same holds, at different frequencies, for the ratio $S=qL/\pi$ and the threshold V of EHDI.

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Appendix

Initial equations

For a square wave electric field, at some half-period with $\mathbf{E}_0(t)=\mathbf{E}_0$, and for ψ independent of time, equations (2) and (7) give the relationship

$$V^2=g\left(V_F^2+Ah_KS^2\right)/N \qquad \text{(A1)}$$

where $V(t)=\mathbf{E}_0(t)L$ is the threshold of external voltage of EHDI and

$$g=g\left(S^2\right)=\frac{h_S+S^2}{1+S^2}\frac{1}{h_S+m\frac{S^2(a_3S^2-a_2)}{\eta_1+(\eta_1+\eta_2+a_4)S^2+\eta_2S^4}}$$

$$h_K=K_{11}/K_{33},\ h_S=\sigma_{\parallel}/\sigma_{\perp},$$

$$h_E=\varepsilon_{\parallel}/\varepsilon_{\perp},\ m=(h_S-h_E)/(h_E-1)$$

$$N=N\left(S^2,x\right)=1-\left[1-h_EP\left(S^2\right)g\left(S^2\right)\right]$$

$$\frac{x}{R\left(S^2\right)}\tanh\left[\frac{R\left(S^2\right)}{x}\right]$$

$$P\left(S^2\right)=\left(1+S^2\right)/\left(h_E+S^2\right),\ R\left(S^2\right)=$$

$$\left(h_S+S^2\right)/\left(h_E+S^2\right) \text{ and } x=\varepsilon_{\perp}f/\pi\sigma_{\perp}.$$

The voltage V is a function of $\mathbf{H}$, S and x. For given values of $\mathbf{H}$ and x (given frequency), in order to obtain the voltage V the value of the threshold (V=min), must be $\partial V/\partial S=0$ or $\partial V^2/\partial S^2=0$. If we derive both terms of equation (A1) with $\partial/\partial S^2$, keeping in mind the condition $\partial V^2/\partial S^2=0$, we obtain

$$V^2=\left[g'\left(V_F^2+Ah_KS^2\right)+Ah_Kg\right]/N' \qquad \text{(A2)}$$

for $g'=g'(S^2)=\partial g(S^2)/\partial S^2$ and $N'=N'(S^2,x)=\partial N(S^2,x)/\partial S^2$.

Substituting the term $V_F^2+Ah_KS^2$ from equation (A1) into equation (A2)

$$V^2=Ah_K\frac{-g^2/g'}{N-N'g/g'}. \qquad \text{(A3)}$$

From equations (A1) and (A3)

$$V_F^2=Ah_K\left(-S^2-\frac{g}{g'}\cdot\frac{N}{N-N'g/g'}\right) \qquad \text{(A4)}$$

If we consider that $\frac{dP(S^2)}{dS^2}=\frac{h_E-1}{(h_E+S^2)^2}\sim 0$ and $\frac{dR(S^2)}{dS^2}=\frac{h_E-h_S}{(h_E+S^2)^2}\sim 0$, then

$$N'=h_EPg'\frac{x}{R}\tanh\frac{R}{x} \text{ and } N-N'\frac{g}{g'}=1-\frac{x}{R}\tanh\frac{R}{x}.$$

Accepting also the simplifications $P(S^2)\sim 1$ and

$R(S^2) \sim 1$ we obtain

$$N - N' \frac{g}{g'} = 1 - x \tanh(x^{-1})$$

and equations (A3), (A4) become

$$V^2 = Ah_K \frac{-g^2/g'}{1 - x\tanh(x^{-1})}$$

$$V_F^2 = Ah_K \left[-(S^2 + g/g') - \frac{g^2}{g'} h_E \frac{x\tanh(x^{-1})}{1 - x\tanh(x^{-1})} \right]$$

Defining the function $u(x) = x \tanh(x^{-1})/[1 - x\tanh(x^{-1})]$ we finally obtain

$$V^2 = Ah_K[1 + u(x)](-g^2/g') \tag{A5}$$

$$V_F^2 = Ah_K \left[-(S^2 + g/g') - \frac{g^2}{g'} h_E u(x) \right] \tag{A6}$$

Simplification of $-(S^2+g/g')$ and $(-g^2/g')$

Let the constants

$$\begin{aligned}
\delta_0 &= \eta_1 h_S & m &= (h_S - h_E)/(h_E - 1) \\
\delta_1 &= \eta_1 + (\eta_1 + \eta_2 + \alpha_1) h_S & \varepsilon_1 &= [\eta_1 + (\eta_1 + \eta_2 + \alpha_1)] h_S - \alpha_2 m \\
\delta_2 &= (\eta_1 + \eta_2 + \alpha_1) + \eta_2 h_S & \varepsilon_2 &= [\eta_2 + (\eta_1 + \eta_2 + \alpha_1)] h_S + (\alpha_3 - \alpha_2) m \\
\delta_3 &= \eta_2 & \varepsilon_3 &= \eta_2 h_2 + \alpha_3 m
\end{aligned}$$

where

$$\eta_1 = \frac{1}{2}(\alpha_4 + \alpha_5 - \alpha_2), \ \eta_2 = \frac{1}{2}(\alpha_3 + \alpha_4 + \alpha_6)$$

and the constants

$$\begin{aligned}
c_0 &= \delta_0^2 & f_0 &= \delta_0^2 & d_0 &= \delta_0(\varepsilon_1 - \delta_1) \\
c_1 &= 2\delta_0\delta_1 & f_1 &= 2\delta_0\delta_1 & d_1 &= 2\delta_0(\varepsilon_2 - \delta_2) \\
c_2 &= \delta_1^2 + 2\delta_0\delta_2 & f_2 &= 3\delta_0\delta_2 - \delta_0\varepsilon_2 + \delta_1\varepsilon_1 & d_2 &= 3\delta_0(\varepsilon_3 - \delta_3) + \delta_1\varepsilon_2 - \delta_2\varepsilon_1 \\
c_3 &= 2(\delta_0\delta_3 + \delta_1\delta_2) & f_3 &= 4\delta_0\delta_3 + 2\delta_2\varepsilon_1 - 2\delta_0\varepsilon_3 & d_3 &= 2(\delta_1\varepsilon_3 - \delta_3\varepsilon_1) \\
c_4 &= \delta_2^2 + 2\delta_1\delta_3 & f_4 &= 3\delta_3\varepsilon_1 + \delta_2\varepsilon_2 - \delta_1\varepsilon_3 & d_4 &= \delta_2\varepsilon_3 - \delta_3\varepsilon_2 \\
c_5 &= 2\delta_2\delta_3 & f_5 &= 2\delta_3\varepsilon_2 & & \\
c_6 &= \delta_3^2 & f_6 &= \delta_3\varepsilon_3 & &
\end{aligned}.$$

From the functions $g = g(S^2)$ and $g' = g'(S^2) = \partial g(S^2)/\partial S^2$ we calculate

$$-(S^2 + g/g') = \frac{f_0 + f_1 S^2 + f_2 S^4 + f_3 S^6 + f_4 S^8 + f_5 S^{10} + f_6 S^{12}}{d_o + d_1 S^2 + d_2 S^4 + d_3 S^6 + d_4 S^8}$$

$$-g^2/g' = \frac{c_0 + c_1 S^2 + c_2 S^4 + c_3 S^6 + c_4 S^8 + c_5 S^{10} + c_6 S^{12}}{d_o + d_1 S^2 + d_2 S^4 + d_3 S^6 + d_4 S^8}.$$

Dividing both terms of each fraction by S^8 and setting $j = 1/S^2$

$$-(S^2 + g/g') = S^4 \frac{f_0 j^6 + f_1 j^5 + f_2 j^4 + f_3 j^3 + f_4 j^2 + f_5 j + f_6}{d_0 j^4 + d_1 j^3 + d_2 j^2 + d_3 j + d_4} = S^4 \phi_1(j)$$

$$-g^2/g' = S^4 \frac{c_0 j^6 + c_1 j^5 + c_2 j^4 + c_3 j^3 + c_4 j^2 + c_5 j + c_6}{d_0 j^4 + d_1 j^3 + d_2 j^2 + d_3 j + d_4} = S^4 \phi_2(j).$$

If we analyse the functions $\phi_1(j)$ and $\phi_2(j)$ in a Taylor expansion relative to j and keeping the first three terms we find

$$\phi_1(j) = h_2 + h_1 j + h_0 j^2 \text{ and } \phi_2(j) = e_2 + e_1 j + e_0 j^2$$

where the constants h_2, h_1, h_0, e_2, e_1, e_0 are given by the relations

$$e_2 = c_6/d_4, \quad e_1 = (c_5 d_4 - c_6 d_3)/d_4^2$$

$$e_0 = \left[d_4^2 (c_4 d_4 - c_6 d_2) - d_3 d_4 (c_5 d_4 - c_6 d_3) \right] / d_4^4$$

$$h_2 = f_6/d_4, \quad h_1 = (f_5 d_4 - f_6 d_3)/d_4^2$$

$$h_0 = \left[d_4^2 (f_4 d_4 - f_6 d_2) - d_3 d_4 (f_5 d_4 - f_6 d_3) \right] / d_4^4$$

from which we obtain

$$-(S^2 + g/g') = h_0 + h_1 S^2 + h_2 S^4 \text{ and } -g^2/g' = e_0 + e_1 S^2 + e_2 S^4$$

Linear equations

From equations (A5), (A6), and under the simplifications described we get

$$V^2 = Ah_K(e_0 + e_1 S^2 + e_2 S^4)[1 + u(x)] \tag{A7}$$

$$V_F^2 = Ah_K[y_0(x) + y_1(x) S^2 + y_2(x) S^4] \tag{A8}$$

where $y_i(x) = h_i + e_i h_E u(x) \cdot (i = 0, 1, 2)$.

From equation (A7) we have the solutions $S^2=-e_1/2e_2\pm\left[\alpha+\beta(x)V^2\right]^{\frac{1}{2}}$ for

$$\alpha=\left(e_1^2-4e_0e_2\right)/4e_2^2 \text{ and } \beta(x)=1/Ah_Ke_2[1+u(x)].$$

The value of $-e_1/2e_2$ is several units, $\beta(x)\sim 1$ and α lies between 10 and 20. Generally, the voltage V is greater than 20 V and the value of $\left[\alpha+\beta(x)V^2\right]^{\frac{1}{2}}$ is greater than the absolute value of $-e_1/2e_2$. Thus, the sign (-) must be dropped and

$$S^2=-\frac{e_1}{2e_2}+\left[\alpha+\beta(x)V^2\right]^{\frac{1}{2}}. \tag{A9}$$

In the same way, from (A8) we find the equation

$$S^2=-\frac{y_1(x)}{2y_2(x)}+\left[\gamma(x)+\varepsilon(x)V_F^2\right]^{\frac{1}{2}} \tag{A10}$$

where $\gamma(x)=\frac{y_1(x)^2-4y_0(x)y_2(x)}{4y_2(x)^2}$ and $\varepsilon(x)=\frac{1}{Ah_Ky_2(x)}$.

For any frequency f (or respective value of x), deleting S^2 from equation (A9) and (A10) and defining $k'(x)=\mathrm{d}V/dV_F$, we get

$$k'(x)=\left(\frac{\varepsilon(x)}{\beta(x)}\right)^{\frac{1}{2}}\cdot\frac{\left(1+\frac{\alpha}{\beta(x)V^2}\right)^{\frac{1}{2}}}{\left(1+\frac{\gamma(x)}{\varepsilon(x)V_F^2}\right)^{\frac{1}{2}}}. \tag{A11}$$

Under the arithmetic assumptions described, the conditions $\alpha/\beta(x)V^2\ll 1$ and $\gamma(x)/\varepsilon(x)V_F{}^2\ll 1$ are valid, and for any specific value of x the quantity $k'(x)$ is essentially invariant under changes of V_F or of the respective magnetic field **H**. The threshold voltage V of EHDI is a linear function of the FT voltage V_F of the form

$$V(x,\ V_F)=\lambda'(x)+k'(x)V_F \tag{A12}$$

for

$$\lambda'(x)=\frac{\left(1+\frac{\alpha}{\beta(x)V^2}\right)^{\frac{1}{2}}}{(\beta(x))^{\frac{1}{2}}}\left[\frac{e_1}{2e_2}-\frac{y_1(x)}{2y_2(x)}+\frac{\gamma(x)}{\left(\gamma(x)+\varepsilon(x)V_F^2\right)^{\frac{1}{2}}}-\frac{\alpha}{\left(\alpha+\beta(x)V^2\right)^{\frac{1}{2}}}\right]$$

and because of the identity $e_1/2e_2-y_1(x)/2y_2(x)\equiv 1/y_2(x)$

$$\lambda'(x)=\frac{\left(1+\frac{\alpha}{\beta(x)V^2}\right)^{\frac{1}{2}}}{(\beta(x))^{\frac{1}{2}}}\left[\frac{1}{y_2(x)}+\frac{\gamma(x)}{\left(\gamma(x)+\varepsilon(x)V_F^2\right)^{\frac{1}{2}}}-\frac{\alpha}{\left(\alpha+\beta(x)V^2\right)^{\frac{1}{2}}}\right] \tag{A13}$$

As we can see from equation (A10), and for $\gamma(x)/\varepsilon(x)V_F{}^2\ll 1$, the square S^2 of $S=qL/\pi$ is also a linear function with respect to V_F for any specific frequency f. We now define $k(x)=dS^2/dV_F$. From (A10)

$$k(x)=dS^2/dV_\mathrm{F}=(\varepsilon(x))^{\frac{1}{2}}\Bigg/\left(1+\frac{\gamma(x)}{\varepsilon(x)V_F^2}\right)^{\frac{1}{2}}. \tag{A14}$$

The slope $k(x)$ is also essentially invariant under changes of V_F, and S^2 is a linear function of FT voltage V_F of the form

$$S^2(x,\ V_F)=\lambda(x)+k(x)V_F \tag{A15}$$

for

$$\lambda(x)=-\frac{y_1(x)}{2y_2(x)}+\frac{\gamma(x)}{\left(\gamma(x)+\varepsilon(x)V_F^2\right)^{\frac{1}{2}}}. \tag{A16}$$

Final formulae

From the equations (A9), (A10), the identity $e_1/2e_2-y_1(x)2y_2(x)\equiv 1/y_2(x)$, the relation $\varepsilon(x)=1/Ah_Ky_2(x)$, and equation (A14) we obtain

$$Ah_Kk(x)+V_F=V\frac{(\beta(x))^{\frac{1}{2}}}{(\varepsilon(x))^{\frac{1}{2}}}\frac{\left(1+\frac{\alpha}{\beta(x)V^2}\right)^{\frac{1}{2}}}{\left(1+\frac{\gamma(x)}{\varepsilon(x)V_F^2}\right)^{\frac{1}{2}}}.$$

Multiplying both terms by $k'(x)$ and using the relation (A11)

$$Ah_Kk(x)k'(x)+k'(x)V_F=V\varpi(x), \tag{A17}$$

$$\text{where } \varpi(x)=\left[1+\frac{\alpha}{\beta(x)V^2}\right]\Bigg/\left[1+\frac{\gamma(x)}{\varepsilon(x)V_F^2}\right]$$

From equations (A12), (A17)

$$\lambda'(x)=Ah_Kk(x)k'(x)+[1-\varpi(x)]V. \tag{A18}$$

In spite of the fact that the term $1-\varpi(x)$ in equation (A18) practically equals zero, we cannot consider the product $[1-\varpi(x)]V$ negligible in front of the other terms in equation (A18). We will try to replace $1-\varpi(x)$ with measurable quantities, making some necessary simplifications.

Analysing the denominator of $\varpi(x)=\left[1+\frac{\alpha}{\beta(x)V^2}\right]\Big/\left[1+\frac{\gamma(x)}{\varepsilon(x)V_F^2}\right]$ in a Taylor expansion relative to V_F, one finds

$$1-\varpi(x)\approx\frac{\gamma(x)}{\varepsilon(x)V_F^2}-\frac{\alpha}{\beta(x)V^2}.$$

Deriving both terms relative to V_F, and keeping in mind the condition $d\varpi(x)/dV_F\approx 0$ and the relation $k'(x)=dV/dV_F$, we get

$$\frac{\gamma(x)}{\varepsilon(x)V_F^4}\approx\frac{\alpha\cdot k'(x)^2}{\beta(x)V^4}.$$

From this last relation and equation (A11) (from which we obtain $k'(x)\approx(\varepsilon(x)/\beta(x))^{\frac{1}{2}}$,

$$\frac{(\gamma(x))^{\frac{1}{2}}}{\varepsilon(x)V_F^2}\approx\frac{\alpha^{\frac{1}{2}}}{\beta(x)V^2}\text{ or }1-\varpi(x)\approx\frac{(\gamma(x))^{\frac{1}{2}}((\gamma(x))^{\frac{1}{2}}-\alpha^{\frac{1}{2}})}{\varepsilon(x)V_F^2}$$

Now, if we analyse the functions $\phi_1(j)$ and $\phi_2(j)$ in a Taylor expansion relative to j and keep only the first two terms, we find $\phi_1(j)=h_2+h_1j$, $\phi_2(j)=e_2+e_1j$ and $\alpha=e_1^2/4e_2^2$, $\gamma(x)=y_1(x)^2/4y_2(x)^2$, which means

$$\alpha^{\frac{1}{2}}-(\gamma(x))^{\frac{1}{2}}=\frac{e_1}{2e_2}-\frac{y_1(x)}{2y_2(x)}=\frac{1}{y_2(x)}=Ah_K\varepsilon(x)$$
$$\left(\text{generally }\frac{e_1}{2e_2}>0,\ \frac{y_1(x)}{2y_2(x)}>0\right)$$
$$1-\varpi(x)\approx Ah_K\frac{-(\gamma(x))^{\frac{1}{2}}}{V_F^2}.$$

From equation (A16) and for $\gamma(x)/\varepsilon(x)V_F^2\ll 1$

$$\lambda(x)\approx-\frac{y_1(x)}{2y_2(x)}=-(\gamma(x))^{\frac{1}{2}}\text{ and }1-\varpi(x)\approx Ah_K\frac{\lambda(x)}{V_F^2}.$$

Finally, equation (A18) becomes

$$\lambda'(x)=Ah_K\left[k(x)k'(x)+\frac{\lambda(x)}{V_F^2}V\right].$$

Let the voltages V, V_F take values around V_0. Then $V\approx V_F\approx V_0$, and from $V_0\approx\lambda'(x)+k'(x)V_0$ (equation A12)

$$\lambda'(x)=Ah_K\left[k(x)k'(x)+\frac{\lambda(x)(1-k'(x))}{\lambda'(x)}\right].\qquad\text{(A19)}$$

We now, for any frequency f (or value x), define as $t(x)\equiv dS^2/dV$. From (A9), (A10) we obtain

$$t(x)\equiv dS^2/dV=(\beta(x))^{\frac{1}{2}}\Big/\left(1+\frac{\alpha}{\beta(x)V^2}\right)^{\frac{1}{2}}.\qquad\text{(A20)}$$

From (A14), (A20) and for $\alpha/\beta(x)V^2\ll 1$ and $\gamma(x)/\varepsilon(x)V_F^2\ll 1$, erasing the function $u(x)$, we obtain

$$\frac{1}{k(x)^2}=h_E\frac{1}{t(x)^2}+C.\qquad\text{(A21)}$$

Equations (A19), (A21) are the final formulae for measuring the ratios $h_K=K_{11}/K_{33}$ and $h_E=\varepsilon_\parallel/\varepsilon_\perp$.